MATHEMATICS FOR CS, A15, TEST 2

Name: _____ Student number_____

(1) (3 marks) Represent the number 314.125 in floating point, single precision memory format.

(2) (2.5 marks) Represent 615 and -81 in integer memory format.

(1.5 marks) Show how the computer performs the floating point computation $0.3234 \times 10^{-4} + (0.8896 \times 10^{-1})(0.8740 \times 10^{-2})$, where the mantissas are truncated to four decimal digits. What is the relative error when the result is truncated to four digits in memory?

(3) (2.5 marks) Determine if the argument form

$$\{p \to r\}, \{p \vee q\}, \{\neg q\} \vdash \{r\}$$

is valid or invalid. Include a truth table and a few words explaining how the truth table supports your conclusion.

(4) (2.5 marks) For the two questions below include a truth table and a few words explaining how the truth table supports your conclusion.
a) Prove or disprove that the logical form $p \wedge (q \vee \neg p) \wedge \neg q$ is a contradiction.

b) Prove or disprove the logical equivalence $p \wedge (\neg(q \wedge r)) \equiv (p \wedge \neg q) \vee (p \wedge \neg r)$.

(5) (2.5 marks) Rewrite the following argument in symbolic form and then use a truth table to investigate its validity. Make sure to draw a conclusion.

Either I read a novel or I both lie on the couch and play a computer game.
I read a novel only if I lie on the couch.
Therefore, if I do not play a computer game, then I do not lie on the couch.

(6) (2.5 marks) Consider the argument form $\{p \to q\}, \{q\} \vdash \{p\}$ (Converse Error). Show that this is an invalid argument form and write a short argument in English which is troubled by Converse Error.

b) Consider the argument form $\{p \to q\}, \{\neg q\} \vdash \{\neg p\}$ (Modus Tollens). Show that this a valid argument form and write a short argument in English which uses Modus Tollens.

(7) (1 mark) Translate the statement "There is a smallest natural number" into logical form with quantifiers. Also write the negation of this statement.

(8) (2 marks) For each of the two statements below write and simplify its negation. Next decide if the statement is true or if the negation is true. Be sure to include justification for your decision.

a) $\forall n \in \mathbb{N}, \{n^2 \in \mathbb{N} \ \wedge \ \frac{1}{2n} \notin \mathbb{N}\}$.

b) $\exists k \in \mathbb{Z}$ s.t. $\forall l \in \mathbb{Z}, kl \leq 0$.