

MATHEMATICS FOR CS, A15, TEST 3

Name: _____

Student number _____

(1) (2.5 marks) Consider the sets $B = \{1, 2, 3\}$, $C = \{\emptyset, 3, 5\}$. Determine (list the elements):

a) $B \times C$.

b) $\mathcal{P}(C)$.

c) What is the cardinality of $\mathcal{P}(C \times C)$.

- (2) (3 marks) A discrete mathematics course has 48 students. 30 of these students play computer games daily, 24 party weekly and 13 are failing the course; 10 of the students play computer games and are failing the course, 9 party and are failing the course and 13 students play computer games and party; 43 students play computer games or party or are failing the course.

a) Draw a Venn diagram indicating the numbers in each cell.

b) How many students play computer games, party and are failing the course?

c) How many students do not play computer games and do not party but are failing the course?

- (3) (3 marks) Consider the sets $X = \{k, l, m, n\}$ and $Y = \{k, l, m\}$.
- How many relations from X to Y are there? Explain.

b) How many relations on X are there? Explain.

c) Draw the graph of an equivalence relation on X with three distinct equivalence classes.

d) Draw the graph of one partial order on X which is not a total order.

- (4) (3 marks) a) Consider the relation $R = \text{mod } 6$ on \mathbb{Z} . Prove that R is an equivalence relation, i.e. demonstrate by explicit computations that R is reflexive, symmetric and transitive.

b) Consider the equivalence relations $R = \text{mod } 6$, $S = \text{mod } 27$ and $T = \text{mod } 10$ on \mathbb{Z} . What is the relation $Q = R \cap S \cap T$, explain! What are the equivalence classes of Q ?

- (5) (2 marks) Recall that the Boolean algebra order on the subsets of a given set is given by $A \leq B$ iff $A \subset B$. Draw the the Hasse diagram of the partial order on the subsets of the set $\mathcal{U} = \{0, 1, 2\}$.

- (6) (2 marks) Convert the Boolean expression into its simplest DNForm and then into complete DNForm: $x(y + z)'(xy)' + (x + yz)'$.

- (7) (2.5 marks) Design and simplify as much as possible a logical circuit which for inputs A , B and C produces output Y where

$$A = 00001111, \quad B = 00110011, \quad C = 01010101, \quad Y = 11010101.$$

Draw the circuit.

(8) (3 marks) Consider the matrices

$$A = \begin{pmatrix} -1 & 2 \\ -3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 & 3 \\ 4 & -1 & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 & -2 \\ 0 & 1 \\ -4 & 0 \end{pmatrix}$$

Evaluate the following expressions or explain why the expression is not defined.

a) CB .

b) $A(2B - C^T)$.

c) $BC - A^2$.

(9) (2 marks) a) Compute the determinant

$$\begin{vmatrix} -5 & 3 & -1 \\ 2 & -2 & -4 \\ 1 & 1 & -3 \end{vmatrix}$$

(10) (2 marks) Solve the linear system

$$\begin{aligned} 5x + 2y &= 4 \\ 4x + 5y &= -7 \end{aligned}$$

by inverting the coefficient matrix. No marks will be given if you solve using a different technique.