DIFFERENTIAL EQUATIONS, H15, TEST 1

(1) (3 marks) Find the solution of the initial value problem

$$ty' - 4y = t^6 e^{2t}, \quad y(1) = 0.$$

(2) (3 marks) Find a solution of the initial value problem

$$y' = y^2 - 4, \quad y(0) = 3.$$

and determine the interval on which this solution is defined.

(3) (2.5 marks) Consider the DE

$$\frac{dy}{dt} = (a + y)(2a - y).$$

(i) Consider the three cases $a < 0, a = 0, a > 0$. In each case find the critical points and determine whether each critical point is asymptotically stable, semistable or unstable.

(ii) In each case sketch the directional field of the equation together with several integral curves.

(iii) Draw the bifurcation diagram of this DE, i.e. plot the location of the critical points versus the parameter a.

(4) (2.5 marks) Find an implicit solution for the initial value problem

$$y' = \frac{x - y^3 + y^2 \sin x}{3xy^2 + 2y \cos x}, \quad y(0) = 1.$$

(5) (2 marks) Use Euler's method with stepsize $h = 0.25$ to approximate the solution of the initial value problem

$$y' = y^2 - 4, \quad y(0) = 3$$

on the interval $0 \leq t \leq 1$. Organize your calculations in a table.

Now look at the exact solution you obtained in problem 2 and the interval on which it is defined and explain in a few sentences why the Euler approximation you just computed is meaningless in this case.

(6) (2 marks) Determine the solution of the initial value problem

$$3y'' + 7y' + 2y = 0, \quad y(0) = -3, \; y'(0) = 1$$

and describe the behaviour of the solution as t increases.