DISCRETE MATHEMATICS, H15, TEST 1

(1) (2 marks) Are the following statement forms logically equivalent: $p \vee q \to p$ and $p \vee (\sim p \wedge q)$? Include a truth table and a few words explaining how the truth table supports your answer.

(2) (2 marks) Consider the argument form

$$p \to \sim q,\ q \to \sim p\ \vdash\ p \vee q$$

Use a truth table to determine whether this form of argument is valid or invalid. Include a few words explaining how the truth table supports your answer.

(3) (3 marks) Consider the set of premises $\{\sim p \vee q \to r,\ s \vee \sim q,\ \sim t,\ p \to t,\ \sim p \wedge r \to \sim s\}$ and the conculsion $\sim q$. Use propositional logic inference rules to deduce the conclusion from the premises, giving a reson for each step.

(4) (1 marks) Rewrite the following statement formally. Use variables and include both quantifiers $\forall$ and $\exists$ in your answer.

a) Every rational number can be written as a ratio of two integers.

b) There exists smallest natural number.

(5) (2 marks) For each of the following statements,
(1) write the negations of the two statements
(2) indicate whether the statement is true or the negation is true and briefly justify your answer.

a) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that $x + y = 0$.

b) $\exists a \in \mathbb{Z}$ such that $\forall b \in \mathbb{Z}$, $ab < b$.

(6) (2 marks) Either prove that the following argument is valid or give an interpretation in which it is false.

$$\forall x, P(x)'\ \vdash\ \forall x, P(x) \to Q(x).$$

(7) (3 marks) Using predicate logic, prove that the following argument is valid. Use the predicate symbols shown.

”Every computer science student works harder than somebody, and everyone who works harder than any other person gets less sleep than that person. Maria is a computer science student. Therefore, Maria gets less sleep than someone else.” $C(x), W(x, y), S(x, y), m$.

(8) (2 marks) Prove or disprove: Given any two rational numbers r and s with $r < s$, there is another rational number between r and s.

(9) (3 marks) Prove from first principles that the number $\sqrt{2} + \sqrt{3}$ is irrational.

(10) (2 marks) Prove that $n^3 - n$ is divisible by 3 for every natural number n.

(11) (2 marks) Prove that for any positive integers $\gcd(a, b) = \gcd(a, a + b)$.

(12) (3 marks) Prove that $\varphi(n)$ is even for all $n > 2$.