

VANIER COLLEGE, DEPARTMENT OF MATHEMATICS
LINEAR ALGEBRA, A15, TEST 1

NAME:

STUDENT ID:

(1) (3 marks) Let X be an invertible 2×2 matrix and let

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}$$

Solve the following equation for the matrix X :

$$(A^2 - A^T)X^{-1} = (XAX^{-1})^2$$

(2) (3 marks) a) For what values of k , if any, will the linear system

$$\begin{aligned}x + 4y - 4z &= k \\x + y &= 2 \\2x - y + k^2z &= k^2\end{aligned}$$

- i) have no solution;
- ii) have a unique solution;
- iii) have infinitely many solutions?

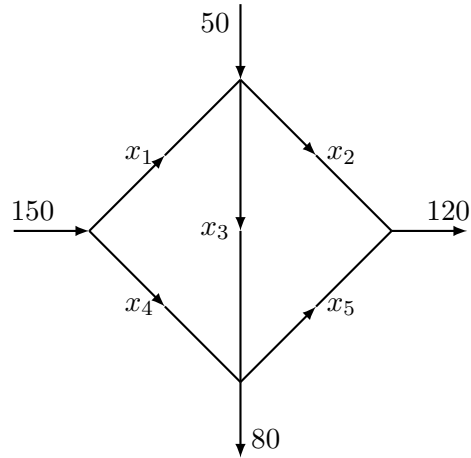
b) For what values of k , if any, is the following matrix M not invertible? Explain.

$$M = \begin{pmatrix} 1 & 4 & -4 \\ 1 & 1 & 0 \\ 2 & -1 & k^2 \end{pmatrix}$$

(3) (2.5 marks) The diagram below represents the flow of traffic in a city block of one way streets.

a) Determine the flow of traffic in the block.

b) Can streets x_2 and x_3 be closed off? What is the traffic flow on x_1 , x_4 and x_5 if this could be done? Is it necessary to reverse the direction of travel on any of x_1 , x_4 or x_5 for this closure?



(4) (3 marks) a) Find the inverse of the matrix

$$C = \begin{pmatrix} -1 & 0 & 1 \\ 1 & 2 & -2 \\ 2 & -1 & 1 \end{pmatrix}$$

b) Now solve the linear system

$$\begin{aligned} -x + z &= -4 \\ x + 2y - 2z &= 3 \\ 2x - y + z &= 1 \end{aligned}$$

(5) (2 marks) True or False? Prove or disprove the following statements. Provide the details of your argument.

a) There exists a 2×4 linear system which is inconsistent.

b) There exists a 4×2 system which has a unique solution.

b) If the linear system $AX = B$ has a solution, then the linear system $(A+I)Z = B$ also has a solution.

- (6) (3 marks) Data has been accumulated on the heights of children relative to their parents. Suppose that the probabilities that a tall parent will have a tall, medium-height or short child are 0.6, 0.2 and 0.2 respectively; the probabilities that a medium height parent will have a tall, medium-height or short child are 0.1, 0.7 and 0.2 respectively; and the probabilities that a short parent will have a tall, medium-height, or short child are 0.2, 0.4 and 0.4, respectively.
- a) Write down the transition matrix for this Markov chain.

b) What is the probability that a short person will have a tall grandchild?

c) Find the proportions of the population consisting of tall, medium-height and short people?

(7) (1.5 marks) a) A 2×3 matrix D can be brought to its reduced row-echelon form

$$RREF(D) = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{pmatrix}$$

by applying three elementary row operations: r_1 , then r_2 and finally r_3 whose corresponding elementary matrices are

$$E_1 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1/2 \end{pmatrix}, E_3 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

Find the matrix D .

(8) (2 marks) a) Suppose that A is a square matrix satisfying the equation

$$A^3 - 5A^2 + 2A + I = 0$$

Prove that A is invertible.

b) If the matrices G and D are such that $RREF(G) = RREF(D)$ prove that there exists an invertible matrix P such that $G = PD$.