

VANIER COLLEGE, DEPARTMENT OF MATHEMATICS
LINEAR ALGEBRA, A15, TEST 2

NAME: _____

STUDENT ID: _____

(1) (3 marks) a) Compute the determinant of

$$B = \begin{pmatrix} 0 & 4 & -1 & 1 \\ -3 & 1 & 1 & 2 \\ 1 & 0 & -2 & 3 \\ 2 & 3 & 0 & 1 \end{pmatrix}$$

b) If a square matrix X obeys the equation

$$2B^T X^{-1} B^{-1} = X B^{-2} X^{-1},$$

where B is the matrix above, use properties of determinants to compute $\det X$.

(2) (3 marks) Compute the adjugate and the inverse of the matrix

$$A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & -1 & 3 \\ -1 & 3 & 1 \end{pmatrix}$$

(3) (3 marks) Solve the linear system

$$\begin{aligned}2x - y + z &= -5 \\x - z &= 2 \\-x + 3y + 2z &= 1\end{aligned}$$

using Cramers Rule.

(4) (3 marks) Consider the two skew lines

$$L_1 = \begin{cases} x = t - 1 \\ y = 2t - 1 \\ z = t - 4 \end{cases}, \quad L_2 = \begin{cases} x = -s + 1 \\ y = 3s + 4 \\ z = -s + 4 \end{cases}$$

Determine the shortest distance between L_1 and L_2 and the points on the two lines closest to each other.

(5) (3.5 marks) Consider the two planes

$$P_1 : 3x - y - z = 5 \quad \text{and} \quad P_2 : x + y - 3z = 4$$

a) Find the equations of the line L of intersection of these two planes.

b) Find the equations of the line L_1 through the point $(1, -1, 2)^T$ and parallel to both planes.

c) Find the shortest distance between the plane P_1 and the line L_1 .

- (6) (3.5 marks) Consider the points $P = (-2, 3, 3)$, $Q = (-4, 0, 5)$ and $R = (1, 5, 2)$.
- a) Determine the lengths of the sides of the triangle PQR .

b) Determine the angles of the triangle PQR .

c) Determine the equation of the plane containing the triangle PQR .

(7) (3.5 marks) Consider the points $A = (-1, -1, 0)^T$, $B = (-1, 3, 1)^T$, $C = (0, 3, 7)^T$ and $D = (2, -4, -1)^T$ in \mathbb{R}^3 .

a) Find the volume of the parallelepiped PRL determined by the points A, B, C, D .

b) Find the area of the face of PRL determined by the points A, C, D .

c) Find the shortest distance between the vertex C and the edge AD .

d) Find the shortest distance between the vertex B and the face determined by the points A, C, D .

(8) (3.5 marks) Consider the point $P = (2, -1, 1)$ and the line $L = \{x = 3 + 2s, y = 10 + 6s, z = 6 - s\}$.

a) Determine the point on the line L closest to P .

b) Determine the shortest distance between the point P and the line L .

c) Determine the equations of the line through the point P and orthogonal and intersecting the line L .